Sigmoidal Functions of the Forms

$$f(x) = \frac{1}{1 + e^{-\alpha x}}$$

$$g(x) = \frac{2}{1 + e^{-\alpha x}} - 1$$

$$h(x) = \tanh(\alpha x)$$

where α is a positive parameter

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NEURAL NETWORKS

(1) Consider the binary sigmoidal function

$$f(x) = \frac{1}{1 + e^{-\alpha x}}$$

Verify that

$$x = \frac{1}{\alpha} \ln \left[\frac{f(x)}{1 - f(x)} \right]$$

(2) A neuron receives inputs 0.5, 1.5, and −1.1 with weights 0.7, 0.9, and 1.2, respectively. If the neuron produces an output signal s of the sigmoidal form

$$s = \frac{1}{1 + e^{-0.6y}}$$

where y is the activation, find the values of y and s. Take the bias weight as 1.3.

(3) A neuron receives inputs 0.6, 1.7, and −1.5 with weights 0.6, 1.1, and 1.3, respectively. It employs a sigmoidal function of the form

$$f(y) = \frac{1}{1 + e^{-\alpha y}}$$

where y is the activation and α is a positive parameter. Find the value of α such that the output signal is 0.66. Take the bias weight as 1.1.

(4) A neuron produces a sigmoidal signal of the form

$$s = \frac{1}{1 + e^{-\alpha y}}$$

where y is the activation and α is a positive parameter. The inputs to the neuron are $x_1 = -0.9$, $x_2 = 0.9$, and $x_3 = 1.2$ with respective weights $w_1 = 0.8$, $w_2 = -0.8$, and $w_3 = 0.4$. Under certain operating conditions, the output signal s is found to be 0.5. Find the value of the bias weight w_0. Comment on the (corresponding) value of α.

(5) For the binary sigmoidal function

$$f(x) = \frac{1}{1 + e^{-\alpha x}}$$

verify that

$$\frac{df(x)}{dx} = \frac{\alpha e^{-\alpha x}}{(1 + e^{-\alpha x})^2}$$

or, alternatively,

$$\frac{df(x)}{dx} = \alpha f(x)[1 - f(x)]$$

(6) Sketch the graph of $\frac{df(x)}{dx}$ vs. $f(x)$ for $\alpha = 0.5$, 1, and 1.5. Show that the maximum value of $\frac{df(x)}{dx}$ is 0.25α and occurs at $f(x) = 0.5$.

(7) A neuron receives two inputs $x_1 = 1.5$ and $x_2 = 1.25$ with weights $w_1 = -1$ and $w_2 = 2$, respectively. The output signal s obeys a sigmoidal function of the form

$$s = \frac{1}{1 + e^{-2y}}$$

where y is the activation. Find the bias weight w_0

when the derivative of s with respect to y is 0.33. What is the corresponding value of s ?

(8) Consider the neural network illustrated in Fig. 1. The inputs are $x_1 = 2$ and $x_2 = -1.5$. The weights (including bias) are

$w_{13} = -1$ $\quad w_{23} = 1.1$ $\quad w_{35} = 1.1$

$w_{14} = -0.5$ $\quad w_{24} = 1.2$ $\quad w_{45} = -1.1$

$w_{03} = 0.7$ $\quad w_{04} = -0.5$ $\quad w_{05} = 0.9$

The two hidden neurons and the output neuron all employ sigmoidal functions of the form

$$f(x) = \frac{1}{1 + e^{-\alpha x}}$$

with $\alpha = 0.8$ for each hidden neuron and $\alpha = 0.6$ for the output neuron. Find the value of the output signal s.

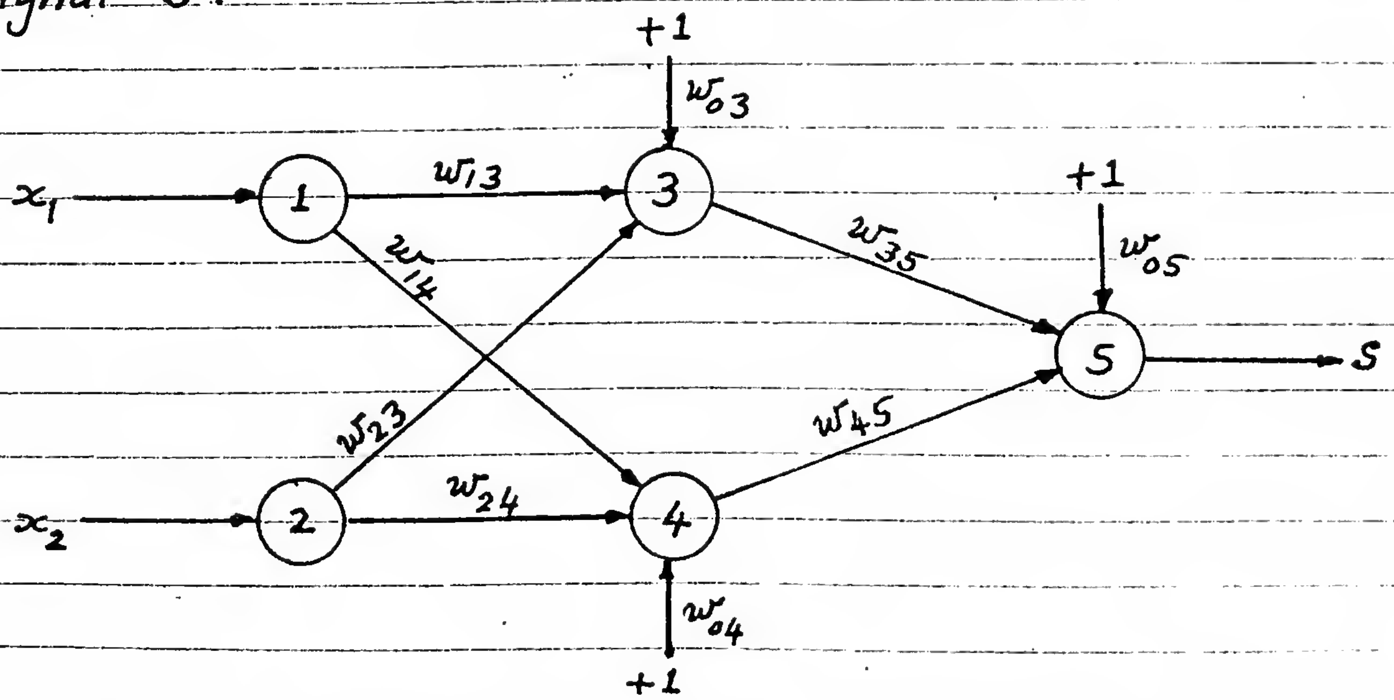

Fig. 1 Neural network for Prob. 8

(9) Consider the bipolar sigmoidal function

$$g(x) = \frac{1 - e^{-\alpha x}}{1 + e^{-\alpha x}}$$

a) Draw, on the same coordinate axes, the graphs of $g(x)$ for $\alpha = 0.5, 1,$ and 2. Comment on these graphs.

b) Verify that

$$x = \frac{1}{\alpha} \ln \left[\frac{1 + g(x)}{1 - g(x)}\right]$$

c) Verify that

$$\frac{dg(x)}{dx} = 0.5\,\alpha\,[1 - g^2(x)]$$

(10) The neuron illustrated in Fig. 2 receives inputs $x_1 = 0.5$, $x_2 = 0.4$, $x_3 = 0.6$ with weights $w_1 = 1.1$, $w_2 = -2.1$, $w_3 = 0.5$ and the bias weight $w_0 = 1.7$. The output signal s is produced according to a sigmoidal function of the form

$$s = \frac{2}{1 + e^{-\alpha y}} - 1$$

where y is the activation. Find the value of the parameter α such that $s = 0.75$.

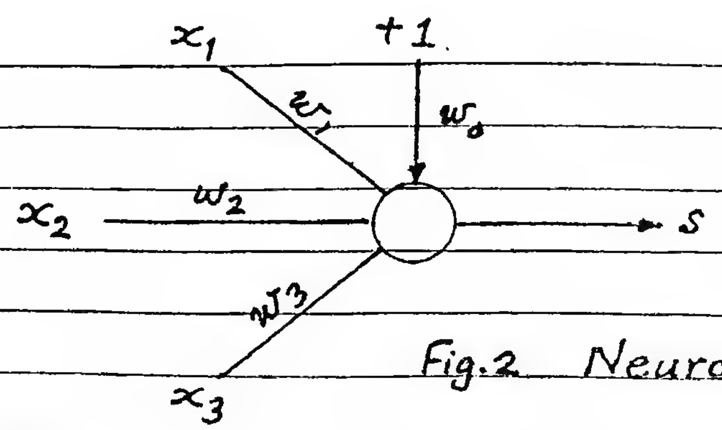

Fig. 2 Neuron for Prob. 10

(11) In Prob. 10, with the value of α arrived at, let the bias weight w_0 be halved in value while all other weights and the inputs are kept unaltered. What is the new value of the output signal s?

(12) In Prob. 11, find the value of the derivative of the output signal s with respect to the activation y.

(13) Show that the bipolar sigmoidal function

$$g(x) = \frac{2}{1+e^{-2x}} - 1$$

is the same as the hyperbolic tangent function $\tanh x$ and that this is a special case of the relationship

$$\frac{2}{1+e^{-\alpha x}} - 1 = \tanh\left(\frac{\alpha x}{2}\right)$$

(14) A neuron receives two inputs $x_1 = 0.7$ and $x_2 = 0.9$ with weights $w_1 = 1.5$ and $w_2 = -1.5$, respectively. The bias weight is $w_0 = 0.8$. If the neuron employs a hyperbolic tangent function, find the value of the output signal.

(15) In Prob. 14, the inputs x_1, x_2 and the weights w_1, w_2 are all kept unchanged while the bias weight w_0 is allowed to change. Find the value of w_0 if the

output signal is to be 0.81.

(16) In Prob. 15, find the value of the derivative of the output signal with respect to the activation.

(17) A neuron employs a hyperbolic tangent function. Under certain operating conditions, the derivative of the output signal s with respect to the activation y is found to be 0.441. Find the values of y and s.

(18) Consider the two-input, two-output neural network illustrated in Fig. 3. All hidden and output neurons employ hyperbolic tangent functions of the form

$$h(x) = \tanh(\alpha x)$$

with $\alpha = 0.5$ for each hidden neuron and $\alpha = 1.5$ for each output neuron. Find the values of the output signals s_1 and s_2.

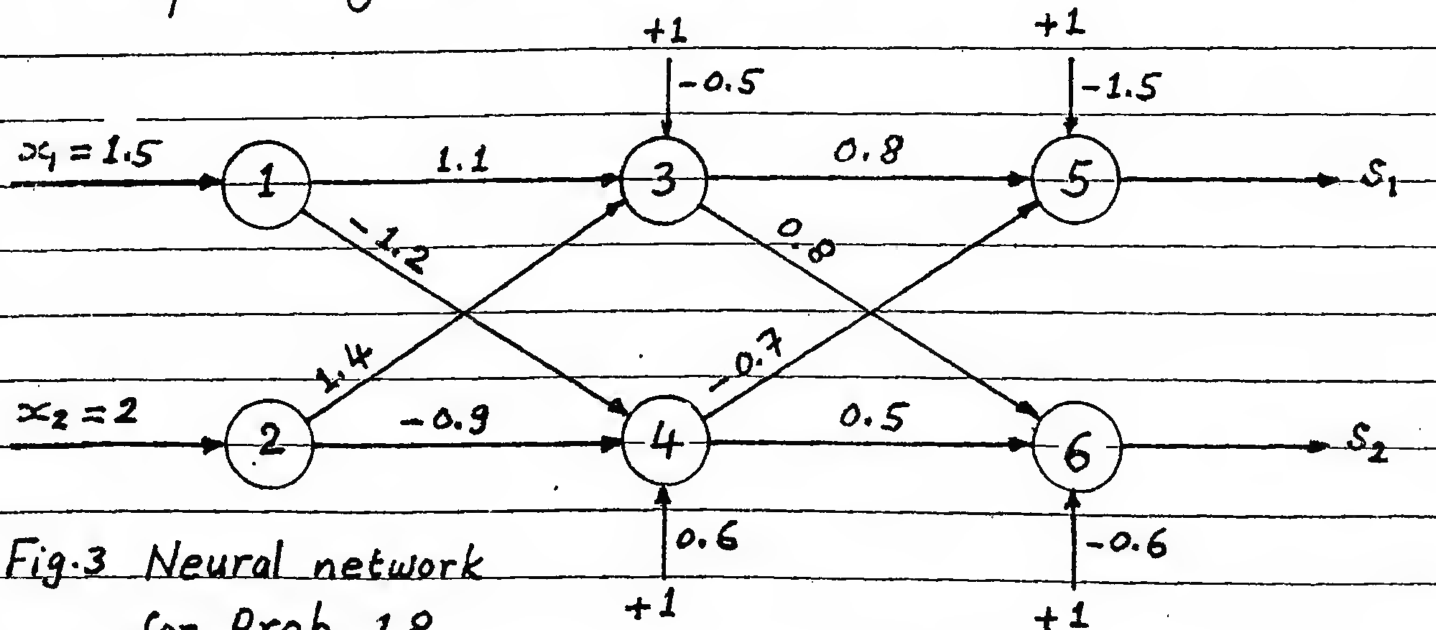

Fig. 3 Neural network for Prob. 18